



Learning Objective

To use spatial visualisation and logical reasoning to solve problems

Materials

cubes, calculator

Focus

These activities allow students to explore the arrangements of cubes and longer shapes in order to determine surface areas and volumes and to investigate the patterns they exhibit. Spatial as well as logical thinking and organisation are involved as students visualise a given shape in terms of its component parts.

Possible difficulties

- Unable to visualise the 3-dimensional solids from the information given
- Does not realise the internal prisms are found by considering the prime factors of the number of cubes
- Does not take into account the dimension of the different cubes when working out volumes
- Does not consider all of the surfaces when working out surface areas

Extension

- Investigate other arrangements of wooden blocks to make steps.
- Reverse the problem with the prisms: if 429 715 or 1001 cubes were used to make a right rectangular prism, which was then painted yellow on the outside, how many cubes would not be painted? What would be the volume and surface area of the unpainted prism?

Teacher Notes for Activity 4



Learning Objective

To use logical reasoning and number sense to solve problems

Materials

counters

Focus

This activity explores problems based on a conceptual understanding of whole numbers and fractions as well as an understanding of what makes sense in the problem contexts. Backtracking from the final position will assist in understanding and solving the problems but counters could also be used to keep track of what is happening. Using a diagram or calculator are other ways to sort through the information while keeping the intent of the problem in mind.

Possible difficulties

- Unable to see how the half-tomato or pumpkin or extra tomatoes or pumpkins fit into the problems
- Simply working on the basis of calculations with the numbers in the problem to obtain incorrect answers

Extension

- Change the numbers in the problems but leave the problem statements the same:
 - More avocados for sale, a different amount more or less than 1 half, 1 third, 3 fourths, ...
 - More avocados or pumpkins at the end of the sale or 3, 4 or 5 more each time.
- Have students make up problems like these of their own and challenge others to solve them using diagrams, tables for try and adjust or algebraic thinking.



Learning Objective

To read, interpret and analyse information

Materials

calculator

Focus

These activities explore concepts of place value, number sense and using data. The relationships among numbers and place value are analysed and students are encouraged to not only find possibilities, but to also disregard numbers and combinations that are not possible. Data needs to be interpreted and analysed to find solutions.

Possible difficulties

- Difficulty with the concepts of fixed cost and variable costs
- Difficulty with the concept of profit and loss
- Poor understanding of place value
- Wanting to add, subtract or multiply rather than using place value or number sense
- Not using all the criteria

Extension

- Students could think up their own profit and loss problems using different criteria.
- Students could make puzzle scroll cards where they take the existing card and change the numbers or context to make a new problem.



Learning Objective

To analyse and determine probability

Materials

calculator

Focus

These activities explore word problems that mostly centre around probability. Students need to determine what the problem is asking and in many cases carry out more than one step in order to find solutions. Analysis of the problems reveals that some problems contain additional information that is not needed. A calculator can be used to assist if necessary as these problems are about reading for information and determining what the problem is asking rather than computation or basic facts.

Possible difficulties

- Confusion over the concepts of chance and probability
- Taking into consideration previous events that are not related to the current event
- Difficulty with the concept of ratio

Extension

- Look at the temperature and number of days of rain for the previous year and explore the probability of rain, sunshine and temperature for your local area.
- Explore the use of vocabulary and how it is used when discussing chance and probability.

Teacher Notes for Activity 9



Learning Objective

To solve problems involving time and make decisions based on particular criteria

Materials

clock

Focus

This activity explores reading for information, obtaining information from a number of sources (the plane information, the timetable, the shuttle bus information) and using it to find solutions. The problems involve thinking about and working with time. Decisions about time being too early or too late are needed rather than an exact time.

Possible difficulties

- Unfamiliarity with a timetable
- Confusion with 24-hour time
- Including taxi and wait time information in the travel calculations
- Thinking that an exact flight is needed rather than flights that are neither too early nor too late

Extension

- Use the information and timetable with other criteria; for example, if you need to be in Cradle Mountain to meet a friend for a hike before dinner or you want to look around Launceston before heading out to Cradle Mountain.



Learning Objective

To use strategic thinking to solve problems

Materials

grid paper, counters in several different colours

Focus

These activities explore more complex problems in which the most difficult step is to find a way of coming to terms with what the problem is asking. Using a table or diagram to explore the situation will assist in seeing all the conditions that need to be considered.

Possible difficulties

- Not using a diagram or table to come to terms with the problem conditions
- Unable to see how to connect the time cycled to the distance travelled
- Considering only some aspects of the puzzle scrolls

Extension

- Change the numbers and scenarios to write other problems based on the puzzle scrolls.
- Use different speeds and times for the problems on page 22.



Learning Objective

To use spatial visualisation, logical and proportional reasoning and an ability to solve problems with fractions and percentage

Materials

counters, calculator

Focus

These activities explore different ways of visualising the problem situation and analysing the possibilities that make up the whole solution. Logical reasoning is needed, as well as an understanding of measurement concepts of length, perimeter, area and direction. In each situation, materials, diagrams or tables can be used to organise, sort and explore the data.

Possible difficulties

- Immediately think that the slow lift will get there last
- Unable to construct tables or draw diagrams to show the relationships among floors, copies or hours needed and allocation of money

Extension

- Change the speed of the lifts, the time waiting for passengers and the floor numbers.
- Groups of students could write their own problems involving percentage and fractions of money spent on activities and challenge others to solve them.

Teacher Notes for Activity 16



Learning Objective

To use patterns and logical reasoning to determine numbers in spatial arrangements

Materials

counters in two different colours, calculator

Focus

This activity explores understanding of numbers in order to discern patterns for triangular numbers. The use of letters to summarise patterns is adopted to lay a foundation for the thinking used in algebra.

Possible difficulties

- Students may find it difficult to accept and use the algebraic form of notation involving T_2 or S_2
- Unable to see how two triangular numbers form a pattern based on a square number
- Unable to complete the pattern to give Pascal's triangle

Extension

- Investigate other arrangements of counters to give numbers – these forms are called polygonal numbers and extend to pentagonal, hexagonal, etc. numbers. Do any of these numbers occur in the Pascal triangle?
- Is it possible to find a relationship between the triangular or square numbers and other polygonal numbers?
- What would happen if this triangular pattern began with the number 2 rather than 1?
- Find some background information about Pascal, a French mathematician who used the patterns on the triangle of numbers that was later named in his honour.
- Investigate the history of this triangle from the times of the Chinese mathematicians and the way it is used currently in mathematics and in applications.



Learning Objective

To identify and use number understandings

Materials

calculator

Focus

These activities explore solving problems involving number sense, magic squares and logic. Analysis of the problems to locate given information is necessary to find the magic number or the arrangement of numbers. Counters, blocks or a calculator can be used to assist as these problems focus on the concepts of number sense and number logic rather than basic facts.

Possible difficulties

- Considering only rows or columns rather than rows, columns and diagonals in magic squares
- Not thinking strategically when doing the sudoku or the alphametic puzzles

Extension

- Investigate other magic squares, magic numbers and alphametic puzzles.
- Explore sudoku games in magazines, newspapers and on the Internet.
- Try writing alphametic puzzles for the other students to do.



Learning Objective

To analyse and use information in word problems

Materials

calculator

Focus

These activities explore word problems that require a number of operations, including division. The wording has been kept fairly simple to help with the Learning Objective process. Students need to determine what the problem is asking and in many cases write out multiple steps to find solutions.

Possible difficulties

- Confusion over the need to carry out more than one step to arrive at a solution
- Using all the numbers listed in the problems rather than just the numbers needed
- Not thinking in terms of the problem and writing solutions such as 763.3 bins
- Difficulty with the concepts of tonne, discount, percentage and profit

Extension

- Students could write their own problems and give them to other students to solve.



Learning Objective

To use strategic thinking to solve problems

Materials

calculator

Focus

This activity explores problems that require analysis of the connections among the data to determine what needs to be done and whether there is a unique solution. A knowledge of simple percentage is also called on. A process of ‘try and adjust’ can be used; however, reasoning logically about the possibilities and using a table or diagram to organise them will be more productive. These ways of thinking can then be generalised to other complex problems.

Possible difficulties

- Not using a table or diagram to manage the data when using ‘Guess and Check’ method
- Only keeping one condition in mind when there are two aspects to be considered
- Unable to see how to change the given prices to find sums and differences in order to get the price of two of an item
- Unable to use the forms of algebraic thinking that answer the problems more directly

Extension

- Discuss the various methods used by students to solve the problems. Include the ones discussed above. Ask them to solve each problem using a different method from what they used or tried first.
- Encourage students to use an algebraic way of thinking about the relationships among the information. Some students may be able to express this diagrammatically or with symbols.
- Challenge students to change the numbers in the problems so that solutions are still possible.
- Students could also write problems using different contexts and larger numbers for others in the class to try.



Learning Objective

To use spatial visualisation and measurement to solve problems

Materials

paper to fold and visualise areas and perimeters, tangram triangles or similar to see the relationships among triangles and squares, calculator

Focus

These activities explore ideas of area and perimeter, using knowledge of squares and triangles to visualise shapes and determine the lengths of sides and areas from which they are composed. Spatial and logical thinking as well as numerical reasoning and organisation are required to investigate the relationships among the shapes to determine distances and areas.

Possible difficulties

- Unable to visualise smaller shapes within the larger shapes
- Cannot see how areas and perimeters are formed from those of the smaller shapes
- Unable to visualise how the diamond forms half of a larger square

Extension

- Investigate other aspects of the problems; for example, make further squares in the first quilting pattern, colour the triangles differently and work out the new areas and fractions, try other square or diamonds in which the length of the diagonal is known.



Learning Objective

To interpret and organise information in series of interrelated statements and to use logical thinking to find solutions

Focus

These activities explore the concepts of average, distance, payments and interrelated statements within problem situations. Students need to read the problems carefully in order to take into consideration a number of different criteria. Tables and lists can be used to help manage the various criteria.

Possible difficulties

- Not using a table or list to manage the data
- Not understanding average
- Confusion when dealing with approximate times and distances

Extension

- Construct a table to show the running distance and how it varies from month to month.
- Write other problems using the same form of complex reasoning for other students to solve.

Teacher Notes for Activity 33



Learning Objective

To use logical reasoning and an ability to visualise a sequence of events so as to use number patterns to solve problems

Materials

calculator

Focus

This activity explores problems with large amount of data that needs both computation and patterning to determine solutions.

Possible difficulties

- Not seeing the way in which the eggs are distributed from the starting line
- Only using the distances to the eggs and not including the distances back to the start

Extension

- Use smaller or larger number of eggs for the race and vary the distance between eggs.
- Ask students to research the mathematician Karl Gauss and his famous solution.



Learning Objective

To organise data and use number understanding to solve problems

Materials

calculator

Focus

These activities explore problems that call on an ability to carefully analyse the relationships among the data and organise the information gained to keep track of the possibilities. Putting the various interrelated aspects into a table or diagram provides a systematic way of dealing with the overlapping conditions.

Amount	\$220		\$200	\$150
	mother	son	mother + daughter	son + daughter
Try	\$120	\$100	\$80	\$180 – too much
Try	\$130	\$90	\$70	\$160 – too much
Try	\$140	\$80	\$60	\$180 – too little
Try	\$135	\$85	\$65	\$150 – correct

Possible difficulties

- Trying to manage the data without using a table or diagram
- Not considering all aspects and information in the puzzle scrolls

Extension

- Challenge students to work out how many people went to each of recording sessions 1, 2 and 3 of the televised quiz show.
- Change the numbers and scenarios to write other problems based on the puzzle scrolls.



Learning Objective

To use logical reasoning, fractions and measurement to solve problems

Materials

paper to draw diagrams or tables and record times, calculator

Focus

These activities investigate relationships among distance and time expressed as fractions of the distance around a cycling track. Logical thinking and organisation are needed to see how the cyclists progress, keeping track of their positions and determining when they will coincide.

Possible difficulties

- Not able to visualise the distance each person cycles or the time he takes
- Does not use a diagram or table to keep track of the various conditions
- Unable to work confidently with fractions
- Trying to manage the data without using a table or list
- Not considering all aspects and information in the puzzle scrolls

Extension

- Change the times for cycling one lap and passing to give different fractions.
- Change the frequency of the days they rode on the cycle track.
- Change the numbers and scenarios to write other problems based on the puzzle scrolls.
- Challenge students to come up with similar problems of their own.



Learning Objective

To analyse information and use proportional and logical reasoning to solve problems

Materials

calculator

Focus

These activities explore problems that call on an ability to analyse carefully the relationships among the several different items of data and use an understanding of proportional reasoning and numbers to organise the information gained to keep track of the possible answers. Page 83 introduces the idea of putting the various interrelated aspects into a diagram that allows all the overlapping conditions to be visualised and dealt with in a systematic way.

Possible difficulties

- Not seeing how proportion applies to the problems
- Working with the numbers in the problem rather than the differences
- Unable to sort out the layers of overlapping information

Extension

- Seek out other problems using Venn diagrams on the Internet.
- Have students investigate the history of logic problems as well as the works of John Venn and Lewis Carroll (the mathematician author of *Alice in Wonderland*).

Teacher Notes for Activity 43



Learning Objective

To use logical reasoning and measurement to solve problems

Materials

several wooden cylinders, block such as base 10 hundred, calculator

Focus

This activity explores the use of models in coming to terms with problem situations and analysing the possibilities that make up the whole solution. Logical reasoning, as well as an understanding of the circumference concept, is required. Diagrams are also helpful to organise and explore the data.

Possible difficulties

- Unable to see how the obelisk will move a distance along the rollers and also a distance along the ground
- Not seeing that division is required to find the number of times the rollers rotate
- Not realising that the obelisk would not require a whole number of days to reach the Nile and would be there during the 22nd day
- Unable to see how 22 days is 3 weeks and 1 day, hence it must be 1 day after a Friday, or a Saturday, that it reached the Nile

Extension

- Ask how far the obelisk will have travelled after 5, 10 or 15 days.
- Pose similar problems to the one on the page where large granite blocks are moved on rollers with different circumferences, and different distances to reach the Pyramids and other large Egyptian constructions.
- If students are familiar with calculating circumferences using the ratio π , pose similar problems where the radius of the roller is given.
- Change the number of teeth on each of the gear wheels (they need to have factors in common to work).
- Have students investigate the use of rollers, block and tackles and gear levers in other situations; e.g. on cranes, boats, winches.



Learning Objective

To use spatial visualisation, logical reasoning and measurement to solve problems

Materials

counters in different colours, clocks

Focus

These activities explore different ways of visualising the problems and analyse different possible solutions. Logical reasoning, as well as an understanding of measurement (length, perimeter, area and direction), is needed. For each question, materials, diagrams or tables can be used to organise, sort and explore the data.

Possible difficulties

- Unable to visualise the paths taken by the person watering the garden, cyclists or balloonist
- Using only 12-hour and not 24-hour time when considering the correct time
- Thinking that the hands on a clock always line up on the hour
- Including 12 o'clock as a time when the hands line up on a clock
- Unable to draw or interpret diagrams to see the relationships among perimeter, side-length and area

Extension

- The problem involving watering trees from a bucket could include more trees, less water per tree and hence more trees per bucket.
- The cyclists could go from the clubhouse to the bay, then go back to the clubhouse and finally return to the bay.
- Students could write more complex problems involving hot air balloons; clocks that lose or gain time; and area, side length and perimeter for a pentagon, hexagon or other polygons.



Learning Objective

To organise data and use an understanding of numbers to solve problems

Materials

calculator

Focus

This activity explores problems that require an ability to carefully analyse the relationships among the data and use an understanding of numbers to organise the information to keep track of the possible answers. Writing the various interrelated aspects in a table is a very helpful way of approaching the problem and provides a systematic way of dealing with a range of problems that have several overlapping conditions.

Possible difficulties

- Not understanding how pages in a book are numbered
- Not reasoning correctly about the pages with two or three stickers per page
- Confusing the page numbers with the number of stickers to provide an answer of 246

Extension

Extend the problem to other situations where pages are numbered; for example:

- After writing a new novel, the author asked her computer to place page numbers on each page. The last page number was 798. How many digits were needed?
- Her computer crashed when only part way through numbering the pages on her autobiography. It had used 3000 digits: Were there any pages with four-digit numbers? What was the last page it numbered? (52 pages more than page 999 or page 1051.)
- Ask students to write similar problems for books by their favourite authors and give them to each other to solve.



Learning Objective

To analyse information and use proportional reasoning to solve problems

Focus

This activity explores problems that require an ability to carefully analyse the relationships among the data and use an understanding of proportional reasoning to suggest possible answers. Several different pieces of information need to be kept in mind – the total number of days worked, the number of people working and the number of days they work each week – to determine when more or less time would be needed and the number of days and weeks that each would amount to.

Possible difficulties

- Not taking note of the number of days in each working week
- Thinking that the number of days worked needs to be divided by the workers and then multiplied to get a larger number of days for the first problem and a smaller number of days for the second problem

Extension

Extend the problem to other situations where work crew numbers vary:

- What would happen if one of the tilers for the first job was ill and could not work?
- What would happen if the plumber in the second problem was only sick for two weeks? (Four plumbers for two weeks would give 40 working days a week. The remaining days require 20 days for each of the five plumbers or four weeks of work. The job would now take six weeks to complete.)
- For the last problem, if the workers only agreed to work six days per week, then how many workers would be needed to finish the job in time? (11 workers would not be enough. 12 workers would be needed to finish the job in time and the last day would not be a full day's work.)

Teacher Notes for Activity 52



Learning Objective

To use logical reasoning and mathematical understanding to solve problems

Materials

calculator

Focus

This activity explores problems based on a conceptual understanding of averages. Many students move immediately to calculations to determine a total, then divide by the amount of numbers that were totalled. This approach works when the average is of a particular group of numbers or results, but these problems require a careful analysis of what is occurring before using a knowledge of the calculation to process the appropriate numbers.

Possible difficulties

- Not understanding the concept of an average
- Only using the numbers given in the problems
- Not analysing the answers to see if they correctly solve the problem
- Not attempting a solution because of the complexity of the steps or calculations

Extension

- Ask what time Jade needs to beat in the first problem if she needs a qualifying time of less than 72 seconds and has one more race to see if she can lower her average. Will she be able to equal a qualifying time of less than 70 seconds if she has two more races to run?
- Have a discussion on the nature of averages. It is not just a calculation but a typical performance over the range of calculations. Students may wish to look into its origin in maritime law.