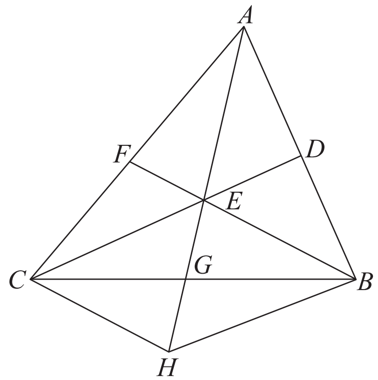
Revision Test 3

Duration: 1 hour

/25

1 Find the equation of the circle that passes through $A(3, 6)$, $B(7, 6)$, $C(7, 3)$ and $D(3, 3)$. [5]

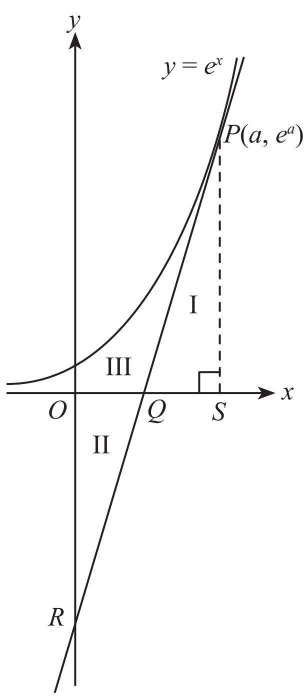
2 In the diagram, F and D are midpoints of AC and AB respectively. BF and CD intersect at E and $AEGH$ is a straight line with E as its midpoint. Prove that G is the midpoint of BC. [5]



3 **(a)** The area of a circle is decreasing at a rate of 4 cm^2/min. Calculate the rate at which the radius of the circle is decreasing when the circumference is 50 cm. [4]

(b) The breadth of a rectangle is increasing at a rate of 2 cm/s and its length is always four times its breadth. At what rate is the area of the rectangle increasing when the breadth is 6 cm? [4]

4 The equation of the curve is $y = e^x$. The point P is (a, e^a) and PQR is a tangent to the curve at P. $S(a, 0)$ is the foot of the perpendicular from P to the x-axis. Given that length of $QS = 1$ unit, find the areas of regions I, II, III in terms of a. [7]



Solutions to Revision Test 3

1 $A(3, 6)$, $B(7, 6)$ and $C(7, 3)$, $D(3, 3)$ are horizontal lines.

$B(7, 6)$, $C(7, 3)$ and $A(3, 6)$, $D(3, 3)$ are vertical lines.

$\therefore$ *ABCD* is a rectangle.

The diagonals of *ABCD* are the diameters of the circle.

We shall use the diagonal *AC*.

$$\text{Midpoint of } AC = \left(\frac{3+7}{2}, \frac{6+3}{2}\right)$$
$$= \left(5, \frac{9}{2}\right)$$

$$\text{Length of } AC = \sqrt{(3-7)^2 + (6-3)^2}$$
$$= \sqrt{16+9}$$
$$= 5$$

Equation of the circle that passes through *ABCD*:

$$(x-5)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{5}{2}\right)^2$$
$$x^2 - 10x + 25 + y^2 - 9y + \frac{81}{4} = \frac{25}{4}$$
$$\mathbf{x^2 + y^2 - 10x - 9y + 39 = 0}$$

2 $AF = FC$ (given)
$AE = EH$ (given)
$\therefore$ *FE* is parallel to *CH*.
$AD = DB$ (given)
$AE = EH$ (given)
$\therefore$ *ED* is parallel to *HB*.
$\therefore$ *CEBH* is a parallelogram.
$CG = GB$ (diagonals to //gram bisect each other)
$\therefore$ *G* is the midpoint of *BC*. **(proven)**

8 **(a)**
$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$
$$-4 = \frac{d(\pi r^2)}{dr} \times \frac{dr}{dt}$$
$$-4 = 2\pi r \times \frac{dr}{dt}$$
$$-4 = C \times \frac{dr}{dt}$$
$$-4 = 50 \times \frac{dr}{dt}$$
$$\frac{dr}{dt} = -\frac{4}{50}$$
$$= -0.08$$

The radius is decreasing at a rate of **0.08 cm/min**.

(b)
$$\frac{dA}{dt} = \frac{dA}{db} \times \frac{db}{dt}$$
$$= \frac{d(lb)}{db} \times \frac{db}{dt}$$
$$= \frac{d}{db}(4b^2) \times 2$$
$$= 8b \times 2$$
$$= 16b$$

When $b = 6$, $\frac{dA}{dt} = 16 \times 6$
$= 96 \text{ cm}^2/\text{s}$

The area is increasing at a rate of **96 cm²/s**.

4
$$\text{Area of region I} = \frac{1}{2} \times QS \times SP$$
$$= \frac{1}{2} \times 1 \times e^a$$
$$= \mathbf{\frac{1}{2}e^a \text{ units}^2}$$

$$OQ = OS - QS$$
$$= a - 1$$

Since $\triangle QSP$ and $\triangle QOR$ are similar,

$$\frac{RO}{PS} = \frac{OQ}{SQ}$$
$$\frac{RO}{e^a} = \frac{a-1}{1}$$
$$RO = e^a(a-1)$$

$$\text{Area of region II} = \frac{1}{2} \times RO \times OQ$$
$$= \mathbf{\frac{1}{2}e^a(a-1)^2 \text{ units}^2}$$

$$\text{Area of region I and III} = \int_0^a e^x \, dx$$
$$= [e^x]_0^a$$
$$= e^a - e^o$$
$$= e^a - 1$$

$$\text{Area of region III} = (e^a - 1) - \text{Area of region I}$$
$$= e^a - 1 - \frac{1}{2}e^a$$
$$= \mathbf{\frac{1}{2}e^a - 1 \text{ units}^2}$$