Revision Test 2

Duration: 1 hour

/25

1 Express $\dfrac{3 - x^2}{(x + 2)(x + 1)}$ as the sum of partial fractions. [4]

2 Find the term independent of x in the sum $\left(x^4 + \dfrac{1}{x^2}\right)^6 + \left(x^2 + \dfrac{1}{x^4}\right)^6$. [5]

3 **(i)** Sketch the graph of $y = |x^2 - 10x + 15|$. [3]

(ii) Draw a straight line on your sketch to show how the roots of $|x^2 - 10x + 15| = 6$ may be formed. [2]

(iii) Solve algebraically the equation $|x^2 - 10x + 15| = 6$. [3]

4 **(a)** Prove that $\dfrac{2\cos^2\theta - 1}{2\sin\theta\cos\theta + 1} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$. [4]

(b) Solve the equation $1 + 2\sin\theta + 2\cos\theta + \tan\theta = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

Solutions to Revision Test 2

1 $\dfrac{3-x^2}{(x+2)(x+1)} = \dfrac{-x^2+3}{x^2+3x+2}$

$$\begin{array}{r} -1 \\ x^2+3x+2 \overline{\big)\, -x^2 \qquad +3} \\ -\quad \underline{-x^2 - 3x - 2} \\ 3x+5 \end{array}$$

$\dfrac{3-x^2}{(x+2)(x+1)} = -1 + \dfrac{3x+5}{(x+2)(x+1)}$

Let $\dfrac{5+3x}{(x+2)(x+1)} = \dfrac{A}{x+2} + \dfrac{B}{x+1}$ where A and B are constants.

$5 + 3x = A(x+1) + B(x+2)$

When $x = -1$, $5 - 3 = B(-1+2)$

$B = 2$

When $x = -2$, $5 - 6 = A(-2+1)$

$-1 = -A$

$A = 1$

$\dfrac{5+3x}{(x-2)(x+1)} = \dfrac{1}{x+2} + \dfrac{2}{x+1}$

$\therefore \dfrac{3-x^2}{(x+2)(x+1)} = \mathbf{\dfrac{1}{x+2} + \dfrac{2}{x+1} - 1}$

2 For $\left(x^4 + \dfrac{1}{x^2}\right)^6$:

$T_{r+1} = \dbinom{6}{r}(x^4)^{6-r}\left(\dfrac{1}{x^2}\right)^r$

$= \dbinom{6}{r}(x^{24-4r})(x^{-2r})$

$\Rightarrow 24 - 4r - 2r = 0$

$6r = 24$

$r = 4$

Term independent of $x = \dbinom{6}{4}(x^4)^2\left(\dfrac{1}{x^2}\right)^4$

$= 15$

For $\left(x^2 + \dfrac{1}{x^4}\right)^6$:

$T_{r+1} = \dbinom{6}{r}(x^2)^{6-r}\left(\dfrac{1}{x^4}\right)^r$

$= \dbinom{6}{r}(x^{12-2r})(x^{-4r})$

$\Rightarrow 12 - 2r - 4r = 0$

$6r = 12$

$r = 2$

Term independent of $x = \dbinom{6}{2}(x^2)^4\left(\dfrac{1}{x^4}\right)^2$

$= 15$

Term independent of x in the sum $= 15 + 15$

$= \mathbf{30}$

3 **(i)** and **(ii)**

For $y = x^2 - 10x + 15 = (x-5)^2 - 10$

When $x = 0$, $y = 15$

When $y = 0$, $(x-5)^2 = 10$

$x - 5 = \pm\sqrt{10}$

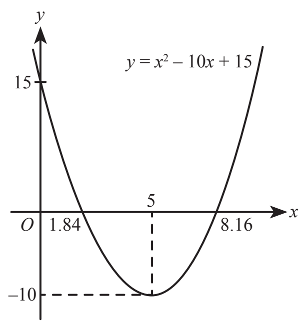
$x = 5 \pm \sqrt{10}$

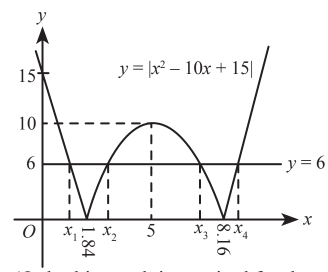
≈ 1.84 or 8.16

Axis of symmetry: $x = 5$

Minimum value of $y = (5-5)^2 - 10 = -10$

Minimum point: $(5, -10)$





(Only this graph is required for the question.)

The roots of $|x^2 - 10x + 15| = 6$ are x_1, x_2, x_3 and x_4.

(iii) $|x^2 - 10x + 15| = 6$

$x^2 - 10x + 15 = -6$ $\qquad$ $x^2 - 10x + 15 = 6$

$x^2 - 10x + 21 = 0$ $\qquad$ $x^2 - 10x + 9 = 0$

$(x-3)(x-7) = 0$ $\qquad$ $(x-1)(x-9) = 0$

$x = 3$ or 7 $\qquad$ $x = 1$ or 9

$\therefore$ $\mathbf{x = 1, 3, 7, 9}$

4 **(a)** $\dfrac{2\cos^2\theta - 1}{2\sin\theta\cos\theta + 1} = \dfrac{\cos 2\theta}{2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{(\cos\theta + \sin\theta)^2}$

$= \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)^2}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$ **(proven)**

(b) $1 + 2\sin\theta + 2\cos\theta + \tan\theta = 0$

$1 + 2\sin\theta + 2\cos\theta + \dfrac{\sin\theta}{\cos\theta} = 0$

$\cos\theta + 2\sin\theta\cos\theta + 2\cos^2\theta + \sin\theta = 0$

$\sin\theta + \cos\theta + 2\sin\theta\cos\theta + 2\cos^2\theta = 0$

$(\sin\theta + \cos\theta) + 2\cos\theta(\sin\theta + \cos\theta) = 0$

$(\sin\theta + \cos\theta)(1 + 2\cos\theta) = 0$

When $\sin\theta + \cos\theta = 0$,

$\sin\theta = -\cos\theta$

$\tan\theta = -1$

$\theta = 135^\circ, 315^\circ$

When $1 + 2\cos\theta = 0$,

$2\cos\theta = -1$

$\cos\theta = -\dfrac{1}{2}$

$\theta = 120^\circ, 240^\circ$

$\therefore$ $\mathbf{\theta = 120^\circ, 135^\circ, 240^\circ, 315^\circ}$