Mid Year Examination Paper 2

INSTRUCTION TO CANDIDATES:

1. Answer **all** questions.
2. Write your answers and working in the spaces provided.
3. Omission of essential working will result in loss of marks.
4. Calculators may be used in this paper.
5. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.

Marks Obtained
/ 50

Duration: 1h 30 min

1 (a) A man jogs 2400 m in 12 min. Express his average speed in kilometres per hour. [2]

(b) Express the ratio 40 cm : 1.5 m in its simplest form. [1]

(c) The temperature inside a cold storage room was –3 ºC when the cooling unit was switched off. The temperature then increases at a constant rate of 0.2 ºC per minute. Find the temperature of the room after 6.5 minutes. [2]

2 (a) Two quantities, m and n, are in inverse proportion such that $m = 7$ when $n = 8$. Find the value of m when $n = 14$. [2]

(b) Two quantities, x^2 and $(y - 2)$, are in direct proportion. Selected values of x and y are shown in the table below.

x	3	2	q
y	20	p	100

(i) Express y in terms of x. [2]

(ii) Hence, find the values of p and q, where p, $q > 0$. [2]

3 Expand and simplify the following expressions.

(a) $2(p + 3q) - [p - 3q - 4(2p - q)]$ [2]

(b) $2(2m + 1)^2 - (2m - 1)^2$ [2]

4 Express the following as a single fraction in its simplest form.

(a) $\dfrac{2}{x+4} - \dfrac{x-1}{x-2}$ [3]

(b) $\dfrac{9a-21}{9a^2-49}$ [3]

5 (a) Simplify $\dfrac{16m^4}{28n^3} \div \dfrac{4m^3}{7n^3}$, giving your answer in the lowest terms. [3]

(b) Hence, simplify $\dfrac{16(p+1)^4}{28n^3} \div \dfrac{4(p+1)^3}{7n^3}$. [1]

6 Solve the following equations.

(a) $6x = \frac{2}{x} + 4$ [2]

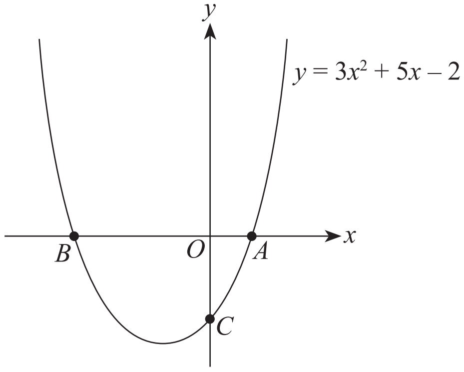
(b) $7x^2 - 2x = 0$ [2]

7 The quadratic graph $y = 3x^2 + 5x - 2$ is shown in the diagram below.

(a) Find the coordinates of the y-intercept, C, of the graph. [1]

(b) By solving the equation $3x^2 + 5x - 2 = 0$, find the coordinates of A and B. [3]

(c) Determine whether the point (1, 5) lies on the graph. Explain your answer. [1]



8 The monthly subscription cost, \$$C$, of Starry Internet Service is given by the formula $C = 25 + \frac{t^2}{100}$ where t is the number of hours used.

(a) Find the amount payable if a customer uses the internet for 16 hours. [1]

(b) What is the minimum cost that a customer must pay every month? [1]

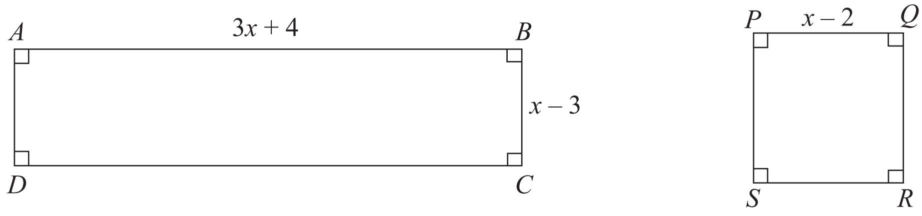
(c) Make t the subject of the above formula and find the number of hours used that month if the subscription paid is \$27.89. [3]

9 The diagrams show rectangle $ABCD$ with dimensions $(3x + 4)$ cm by $(x - 3)$ cm, and a square $PQRS$ with side $(x - 2)$ cm. The area of the rectangle is 4 times the area of the square.

(a) Find and simplify expressions in terms of x for the areas of rectangle $ABCD$ and the square $PQRS$ respectively. [2]

(b) Form an equation in x and show that it simplifies to become $x^2 - 11x + 28 = 0$. [2]

(c) Find the area of the square, given that $x > 4$. [2]



10 ***Answer the following question on the graph paper provided.***

(a) Using a scale of 2 cm to represent 1 unit on both the x and y axes, draw the graphs of the following equations for $-1 \leqslant x \leqslant 3$.

(i) $y = -2x + 2$ [2]

(ii) $x - 2y = 6$ [2]

(b) Hence, solve the following simultaneous equations $y = -2x + 2$ and $x - 2y = 6$ graphically. [1]

Solutions to:

Mid Year Examination Paper 2

1. (a) $\text{Speed} = \dfrac{2400 \text{ m}}{12 \text{ min}}$

$= \dfrac{200 \text{ m}}{1 \text{ min}}$

$= \dfrac{0.2 \text{ km}}{\frac{1}{60} \text{ h}}$

$= 12 \text{ km/h}$

(b) $40 \text{ cm} : 1.5 \text{ m} = 40 \text{ cm} : 150 \text{ cm}$

$= 4 : 15$

(c) Final temperature

$= -3 + 6.5(0.2)$

$= -1.7\ ^\circ\text{C}$

2. (a) <u>Method 1</u>

Since m and n are in inverse proportion, mn is a constant.

$\therefore 7(8) = m(14)$

$56 = 14m$

$m = 4$

<u>Method 2</u>

$m = \dfrac{k}{n}$, where k is a constant.

$7 = \dfrac{k}{8}$

$k = 56$

$\therefore m = \dfrac{56}{n}$

When $n = 14$,

$m = \dfrac{56}{14}$

$= 4$

(b) (i) $y - 2 = kx^2$

When $x = 3, y = 20$,

$20 - 2 = k(3^2)$

$18 = 9k$

$k = 2$

$\therefore y - 2 = 2x^2$

$\Rightarrow y = 2x^2 + 2$

(ii) When $x = 2, y = p$

$p = 2(2^2) + 2$

$= 10$

When $x = q, y = 100$,

$100 = 2q^2 + 2$

$2q^2 = 98$

$q^2 = 49$

$q = \pm\sqrt{49}$

$= 7$ or -7 (rej.)

3. (a) $2(p + 3q) - [p - 3q - 4(2p - q)]$

$= 2(p + 3q) - (p - 3q - 8p + 4q)$

$= 2p + 6q - (-7p + q)$

$= 2p + 6q + 7p - q$

$= 9p + 5q$

(b) $2(2m + 1)^2 - (2m - 1)^2$

$= 2(4m^2 + 4m + 1) - (4m^2 - 4m + 1)$

$= 8m^2 + 8m + 2 - 4m^2 + 4m - 1$

$= 4m^2 + 12m + 1$

4. (a) $\dfrac{2}{x+4} - \dfrac{x-1}{x-2} = \dfrac{2(x-2) - (x-1)(x+4)}{(x+4)(x-2)}$

$= \dfrac{2x - 4 - (x^2 + 3x - 4)}{(x+4)(x-2)}$

$= \dfrac{2x - 4 - x^2 - 3x + 4}{(x+4)(x-2)}$

$= \dfrac{-x^2 - x}{(x+4)(x-2)} \quad \left(\text{or } -\dfrac{x^2 + x}{(x+4)(x-2)}\right)$

(b) $\dfrac{9a - 21}{9a^2 - 49} = \dfrac{3\cancel{(3a-7)}}{\cancel{(3a-7)}(3a+7)}$

$= \dfrac{3}{3a+7}$

5. (a) $\dfrac{16m^4}{28n^3} \div \dfrac{4m^3}{7n^3} = \dfrac{16m^{\cancel{4}}}{28\cancel{n^3}} \times \dfrac{7\cancel{n^3}}{4\cancel{m^3}}$

$= \dfrac{\overset{4}{\cancel{16}}m}{\underset{4}{\cancel{28}}} \times \dfrac{\cancel{7}}{\cancel{4}}$

$= \dfrac{4m}{4}$

$= m$

(b) By letting $p + 1 = m$, the two equations are similar.

Hence from (a),

$\dfrac{16(p+1)^4}{28n^3} \div \dfrac{4(p+1)^3}{7n^3} = p + 1$

6. (a) $6x = \dfrac{2}{x} + 4$

$6x = \dfrac{2 + 4x}{x}$

$6x^2 = 2 + 4x$

$6x^2 - 4x - 2 = 0$

$(3x + 1)(2x - 2) = 0$

$3x + 1 = 0$ or $2x - 2 = 0$

$\therefore x = -\dfrac{1}{3}$ or $x = 1$

×	$2x$	-2
$3x$	$6x^2$	$-6x$
1	$2x$	-2

(b) $7x^2 - 2x = 0$

$x(7x - 2) = 0$

$\therefore x = 0$ or $7x - 2 = 0$

$7x = 2$

$x = \dfrac{2}{7}$

7. (a) At y-intercept C, hence $x = 0$.

$y = 3(0)^2 + 5(0) - 2 = -2$

$\therefore C\ (0, -2)$

(b) $3x^2 + 5x - 2 = 0$

$(3x - 1)(x + 2) = 0$

$3x - 1 = 0$ or $x + 2 = 0$

$\therefore x = \frac{1}{3}$ or $x = -2$

╳	x	2
$3x$	$3x^2$	$6x$
-1	$-x$	-2

$\therefore A\left(\frac{1}{3}, 0\right)$ and $B(-2, 0)$

(c) Substituting $x = 1$ and $y = 5$ into $y = 3x^2 + 5x - 2$,

LHS = 5

RHS $= 3(1)^2 + 5(1) - 2$

$= 6$

Since LHS $\neq$ RHS, (1, 5) does not lie on the graph.

8. (a) $C = 25 + \frac{16^2}{100}$

$= \$27.56$

(b) When $t = 0$, $C = 25 + \frac{0^2}{100}$

$= 25$

The minimum cost is \$25.

(c) $C = 25 + \frac{t^2}{100}$

$C - 25 = \frac{t^2}{100}$

$100C - 2500 = t^2$

$t = \sqrt{100C - 2500}$ or $-\sqrt{100C - 2500}$ (rej. $\because t > 0$)

$\therefore$ when $C = \$27.89$,

$t = \sqrt{100(27.89) - 2500}$

$= 17$ hours

9. (a) Area of square $= (x - 2)^2$

$= (x^2 - 4x + 4)\text{ cm}^2$

Area of rectangle $= (3x + 4)(x - 3)$

$= 3x^2 - 9x + 4x - 12$

$= (3x^2 - 5x - 12)\text{ cm}^2$

(b) Area of rectangle = 4(Area of square)

$3x^2 - 5x - 12 = 4(x^2 - 4x + 4)$

$3x^2 - 5x - 12 = 4x^2 - 16x + 16$

$x^2 - 11x + 28 = 0$ (shown)

(c) $x^2 - 11x + 28 = 0$

$(x - 7)(x - 4) = 0$

$x - 7 = 0$ or $x - 4 = 0$

$\therefore x = 7$ or $x = 4$ (rej. $\because x > 4$)

Area of square $PQRS = (7 - 2)^2$

$= 25\text{ cm}^2$

10. (a) (i) $y = -2x + 2$

x	-1	0	3
$y = -2x + 2$	4	2	-4

(ii) $x - 2y = 6$

$\Rightarrow 2y = x - 6$

$\Rightarrow y = \frac{1}{2}x - 3$

x	-1	0	3
$y = \frac{1}{2}x - 3$	-3.5	-3	-1.5

(see diagram 1 on page S3)

(b) Hence, $x = 2$, $y = -2$.

Diagram 1