

Mid Year Examination Paper 2

INSTRUCTION TO CANDIDATES:

1. Answer **all** questions.
2. Write your answers and working in the spaces provided.
3. Omission of essential working will result in loss of marks.
4. Calculators may be used in this paper.
5. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.

Marks Obtained
80

Duration: 2 hours

- 1** (a) Solve the following equations.
- (i) $3x^2 - 6x = 15x + x^2$ [3]
- (ii) $\frac{6}{p-1} = \frac{p+1}{4}$ [3]
- (b) Given that $\frac{1}{m} + \frac{1}{n} = \frac{1}{q+3}$, express q in terms of m and n . [4]
- (c) Find the possible values of z in the equation $(z+3)^z = 1$. [2]

- 2** Three sons Andrew, Benjamin and Charlie inherited a sum of money in the ratio 4 : 3 : 7 respectively.
- (a) If Charlie received SGD \$92 000 more than Benjamin, calculate the amount of money that Charlie received. [2]
- (b) Charlie decided to invest his sum of money in a bank. Two bank packages were offered to him as shown in the table below.

Bank A	Bank B
Simple interest rate = 2.6% per annum \$4000 bonus per year.	Compound interest rate = 2.4% per annum No yearly bonus.

By showing your working clearly, explain which bank should Charlie invest his money in if he wants a better return after 3 years. [4]

- (c) Andrew converts his Singapore dollars to US dollars at the rate of USD 1 = SGD 1.32. Find his profit earned when he converts the US dollars back to Singapore dollars at a new rate of US 1 = SGD 1.40 the following year. Give your answer in SGD. [3]

3 (a) Simplify the following expressions, expressing your answers with positive indices.

(i) $\sqrt[3]{\sqrt{64m^{12}}}$ [2]

(ii) $\frac{16p^{\frac{1}{2}} \times 2p^{\frac{3}{4}} \div 12p^{-\frac{5}{2}}}{6p^{\frac{2}{3}}}$ [3]

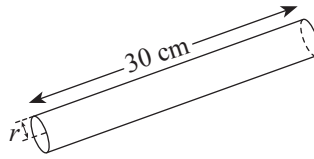
(b) Find the value of p in the equation $7^{p+1} \times \frac{1}{49^{7-p}} = 343^{\frac{1}{3}}$. [3]

(c) One microgram, $1 \mu\text{g}$, is 10^{-6} times of one gram.

(i) Express $2.31 \times 10^6 \mu\text{g}$ in grams, in standard form. [1]

(ii) 100 g of a chocolate drink contains $2.31 \times 10^6 \mu\text{g}$ of fats. Calculate the mass of fats in grams, in 2 kg of the chocolate drink, expressing your answer in standard form. [3]

- 4 The figure below shows a solid silver cylindrical rod with radius r cm and length 30 cm. The surface area of the rod is 300 cm^2 .



- (a) Form an equation in terms of π and r , and show that it reduces to

$$r^2 + 30r - \frac{150}{\pi} = 0. \quad [3]$$

- (b) By taking $\pi = 3.142$, solve the equation above to find the radius of the rod. Give your answer correct to 2 decimal places. [4]

- (c) Hence, find the mass of the entire rod, given that the density of silver is 10.5 g/cm^3 . [2]

- 5** A factory needs to produce 52 toy cars every day. On Day 1, the factory manufactured the toy cars at a rate of x cars per hour. On Day 2, due to some technical problems, the factory's manufacturing speed was 3 cars/hour less.
- (a) Write down an expression in x for the number of hours the factory took to manufacture 52 cars on Day 1. [1]
- (b) Write down an expression in x for the number of hours the factory took to manufacture 52 cars on Day 2. [1]
- (c) Given that the difference in both duration was 1 hour, form an equation in x and show that it reduces to $x^2 - 3x - 156 = 0$. [3]
- (d) A factory supervisor is paid an hourly rate of \$15/hour during the production of the toy cars. Calculate his pay earned on Day 1, correct to the nearest cent. [3]

6 (a) Solve the inequality $\frac{4x-1}{10} \leq \frac{x}{2} < 20 - 2x$ and show your answer on a number line. [4]

(b) Mr Lim and Mrs Lim decide to save money in a joint bank account. Mrs Lim deposited $\$x$ yearly in this account, while Mr Lim's annual deposit is $\$2100$ more than twice the amount of Mrs Lim's annual deposit.

(i) Write down an expression in terms of x , for the amount of money deposited by Mr Lim in a year. [1]

(ii) Form an inequality in terms of n and x , if they are able to purchase a car which cost $\$65\,160$ after n years. [2]

(iii) Given that they need 6 years in order to purchase the car, solve the inequality and find the minimum amount that Mrs Lim must save yearly. [3]

7 The diagram shows $\triangle ABC$ such that DE is parallel to BC . It is given that $AD = 4$ cm, $BN = 7$ cm and $BC = BD = 8$ cm.

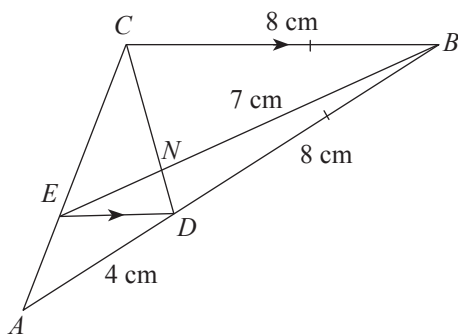
(a) Prove that $\triangle DNE$ is similar to $\triangle CNB$. [3]

(b) Find the length of NE . [3]

(c) Write down the value of

(i) $\frac{\text{area of } \triangle DNE}{\text{area of } \triangle CNB}$, [1]

(ii) $\frac{\text{perimeter of } \triangle ADE}{\text{perimeter of } \triangle ABC}$. [1]



- 8 In diagram 1, a funnel is in the shape of an inverted cone with the bottom conical section ABX cut off. The radius of the larger base with centre O is 12 cm while the radius of the cut-off conical section is 3 cm. The height of the funnel, ON , is 10 cm.

In diagram 2, the funnel is attached to a cylinder of base radius 3 cm. The funnel is filled completely with water. The water is then allowed to flow completely into the cylinder.

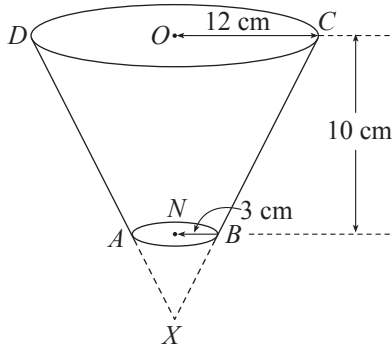


Diagram 1

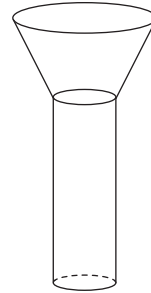


Diagram 2

- (a) Using similar triangles, find the height NX of the cone AXB . [3]
- (b) Calculate the height OX of the cone DXC . [2]
- (c) Hence, find
- (i) the volume of the funnel in terms of π , [4]
- (ii) the height of water in the cylinder after the water has completely flowed into it. [3]
- (Vol. of cone = $\frac{1}{3}\pi r^2 h$)

Solutions to:

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1. (a) (i) $3x^2 - 6x = 15x + x^2$
 $3x^2 - 6x - 15x - x^2 = 0$
 $2x^2 - 21x = 0$
 $x(2x - 21) = 0$
 $x = 0$ or $2x - 21 = 0$
 $x = 10\frac{1}{2}$

(ii) $\frac{6}{p-1} = \frac{p+1}{4}$
 $24 = (p-1)(p+1)$ cross-multiply
 $24 = p^2 - 1$ (a-b)(a+b) = a^2 - b^2
 $p^2 = 25$
 $p = \pm\sqrt{25}$
 $= -5$ or 5

(b) $\frac{1}{m} + \frac{1}{n} = \frac{1}{q+3}$
 $\frac{n+m}{mn} = \frac{1}{q+3}$
 $(n+m)(q+3) = mn$ cross-multiply
 $nq + 3n + mq + 3m = mn$
 $nq + mq = mn - 3n - 3m$ terms with q on LHS
 $q(n+m) = mn - 3n - 3m$ factorise q
 $q = \frac{mn - 3n - 3m}{n+m}$

(c) $(z+3)^z = 1$
 Either $z+3 = 1$ or $z = 0$
 (since $1^a = 1$) (since $a^0 = 1$)
 $\therefore z = -2$ or $z = 0$

2. (a) Amount received by Charlie
 $= \frac{7}{7-3} \times \$92\,000$
 $= \$161\,000$

(b) **Bank A**
 Interest received $= \frac{161\,000 \times 2.6 \times 3}{100}$ I = $\frac{PRT}{100}$
 $= \$12\,558$

Total amount received
 $= \$12\,558 + \$161\,000 + \$ (4000 \times 3)$
 $= \$185\,558$ include annual bonus

Bank B
 Total amount received
 $= 161\,000 \left(1 + \frac{2.4}{100}\right)^3$ $A_n = P\left(1 + \frac{r}{100}\right)^n$
 $= \$172\,872.43$ (2 d.p.)

Hence, Charlie should invest in Bank A, since he receives more returns.

(c) Amount received by Andrew
 $=$ Difference between Charlie and Benjamin 4 units
 $= \$92\,000$

At USD 1 = SGD 1.32,
 Andrew's money in USD
 $=$ SGD $\$92\,000 \div 1.32$
 $=$ USD $\$69\,696.97$ (nearest cent)

At USD 1 = SGD 1.40,
 Andrew's money after conversion to SGD
 $=$ USD $69\,696.97 \times 1.4$
 $=$ SGD $\$97\,575.76$ (nearest cent)

Profit earned $=$ $\$97\,575.76 - \$92\,000$
 $=$ $\$5575.76$

3. (a) (i) $\sqrt[3]{64m^{12}} = \sqrt[3]{(64m^{12})^{\frac{1}{3}}}$
 $= \sqrt[3]{8m^6}$ $\sqrt[3]{64} = 8$
 $= 2m^2$

(ii) $\frac{16p^{\frac{1}{2}} \times 2p^{\frac{3}{4}} \div 12p^{-\frac{5}{2}}}{6p^{\frac{2}{3}}} = \frac{32p^{\frac{1}{2} + \frac{3}{4}}}{12p^{-\frac{5}{2}} \times 6p^{\frac{2}{3}}}$
 $= \frac{8p^{\frac{5}{4}}}{3p^{-\frac{5}{2}} \times 6p^{\frac{2}{3}}}$
 $= \frac{4p^{\frac{5}{4}}}{9p^{-\frac{5}{2} + \frac{2}{3}}}$
 $= \frac{4}{9}p^{\frac{5}{4} - (-\frac{5}{2} + \frac{2}{3})}$
 $= \frac{4}{9}p^{\frac{37}{12}}$

(b) $7^{p+1} \times \frac{1}{49^{7-p}} = 343^{\frac{1}{3}}$
 $7^{p+1} \times \frac{1}{7^{14-2p}} = (7^3)^{\frac{1}{3}}$
 $7^{p+1-(14-2p)} = 7^1$
 $7^{p+1-14+2p} = 7^1$
 $7^{3p-13} = 7^1$
 $3p - 13 = 1$ comparing indices
 $3p = 14$
 $p = 4\frac{2}{3}$

(c) (i) $2.31 \times 10^6 \mu\text{g} = (2.31 \times 10^6) \times 10^{-6} \text{ g}$
 $= 2.31 \text{ g}$

(ii) 100 g of chocolate drink contains 2.31 g fat.
 Mass of fat in 2 kg of chocolate drink
 $= \frac{2.31}{100} \times 2000$
 $= 46.2 \text{ g}$
 $= 4.62 \times 10 \text{ g}$

4. (a) $2\pi r^2 + 2\pi r h = 300$
 $2\pi r^2 + 2\pi r(30) = 300$
 $2\pi r^2 + 60\pi r = 300$
 $2(\pi r^2 + 30\pi r) = 300$
 $\pi r^2 + 30\pi r = 150$
 $\pi r^2 + 30\pi r - 150 = 0$
 $r^2 + 30r - \frac{150}{\pi} = 0$ (shown) — ①

(b) From ①,
 $a = 1, b = 30, c = -\frac{150}{\pi}$
Using the quadratic formula,
 $r = \frac{-30 \pm \sqrt{30^2 - 4(1)(-\frac{150}{\pi})}}{2(1)}$ $-\frac{150}{\pi} \approx -47.74$

≈ 1.515 or -31.515 (rej.)
Hence, $r = 1.52$ (2 d.p.)
The radius of the rod is 1.52 cm.

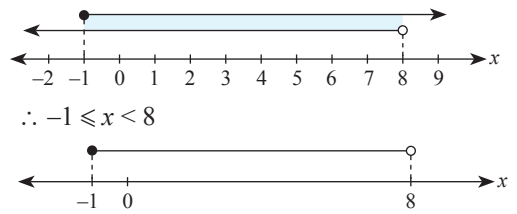
(c) Volume of rod
 $= \pi r^2 h$
 $= 3.142 \times (1.515)^2 \times 30$ use $r \approx 1.515$ for accuracy
 $\approx 216.3479 \text{ cm}^3$
Mass of rod = density \times volume
 $= 10.5 \times 216.347$
 $= 2271.635 \text{ g}$
 $= 2270 \text{ g}$ (3 s.f.)

5. (a) No. of hours $= \frac{1}{x} \times 52$
 $= \frac{52}{x} \text{ h}$
(b) No. of hours $= \frac{1}{x-3} \times 52$
 $= \frac{52}{x-3} \text{ h}$
(c) $\frac{52}{x-3} - \frac{52}{x} = 1$
 $\frac{52x - 52(x-3)}{x(x-3)} = 1$
 $52x - 52x + 156 = x^2 - 3x$
 $x^2 - 3x - 156 = 0$ (shown) — ①

(d) From ①,
 $a = 1, b = -3, c = -156$
Using the quadratic formula,
 $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-156)}}{2(1)}$
 $= 14.0797$ or -11.0797 (rej.)

No. of hours on Day 1 $= \frac{52}{14.079}$
 $\approx 3.6934 \text{ h}$
Supervisor's pay $= \$15 \times 3.6934$
 $= \$55.40$ (nearest cent)

6. (a) $\frac{4x-1}{10} \leq \frac{x}{2} < 20 - 2x$
 $\frac{4x-1}{10} \leq \frac{x}{2}$ and $\frac{x}{2} < 20 - 2x$
 $4x - 1 \leq 5x$ $x < 40 - 4x$
 $x \geq -1$ $5x < 40$
 $x < 8$



(b) (i) Money deposited by Mr Lim in a year
 $= \$(2x + 2100)$
(ii) Total amount deposited in a year
 $= \$(2x + 2100 + x)$
 $= \$(3x + 2100)$
Since they can afford a car after n years,
the required inequality is:
 $n(3x + 2100) \geq 65\,160$ — ①
(iii) Subst. $n = 6$ into ①,
 $6(3x + 2100) \geq 65\,160$
 $18x + 12\,600 \geq 65\,160$
 $18x \geq 52\,560$
 $x \geq 2920$
The minimum amount that Mrs Lim must
save yearly is \$2920.

7. (a) $\angle EDN = \angle BCN$ (alt. \angle s, $DE \parallel BC$)
 $\angle DEN = \angle CBN$ (alt. \angle s, $DE \parallel BC$)
 $\angle END = \angle BNC$ (vert. opp. \angle s)
Hence, $\triangle DNE$ is similar to $\triangle CNB$, by AAA.
(b) $\triangle EAD$ is similar to $\triangle CAB$. prove it!
Hence, $\frac{AD}{AB} = \frac{DE}{CB}$
 $\frac{4}{12} = \frac{DE}{CB}$ — ①
Also, $\frac{NE}{NB} = \frac{DE}{CB}$
 $\frac{NE}{7} = \frac{4}{12}$ from ①
 $NE = 2\frac{1}{3} \text{ cm}$

(c) (i) By similarity,
 $\frac{\text{Area of } \triangle DNE}{\text{Area of } \triangle CNB} = \left(\frac{1}{3}\right)^2$ $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$
 $= \frac{1}{9}$
(ii) $\frac{\text{Perimeter of } \triangle ADE}{\text{Perimeter of } \triangle ABC} = \frac{1}{3}$

8. (a) $\triangle BNK$ is similar to $\triangle COX$. prove it!
Hence, $\frac{NB}{OC} = \frac{NX}{XO}$
 $\frac{3}{12} = \frac{NX}{XO}$
 $\frac{NX}{XO} = \frac{1}{4}$ — ①

From ①,

NX rep. 1 unit;

OX rep. 4 units;

NO rep. 3 units;

\therefore Height of cone $AXB = \frac{1}{3} \times 10$

$$= 3\frac{1}{3} \text{ cm}$$

(b) $OX = \frac{4}{3} \times 10$ $\therefore OX$ rep. 4 units

$$= 13\frac{1}{3} \text{ cm}$$

(c) (i) Volume of cone $DXC = \frac{1}{3}\pi(OC^2)(XO)$

$$= \frac{1}{3}\pi(12)^2\left(13\frac{1}{3}\right)$$

$$= 640\pi \text{ cm}^3$$

Volume of cone $AXB = \frac{1}{3}\pi(NB^2)(NX)$

$$= \frac{1}{3}\pi(3)^2\left(3\frac{1}{3}\right)$$

$$= 10\pi \text{ cm}^3$$

Volume of funnel $= 640\pi - 10\pi$

$$= 630\pi \text{ cm}^3$$

(ii) Height of water in the cylinder

$$= \frac{\text{Volume of funnel}}{\text{Base area of cylinder}}$$

$$= \frac{630\pi}{\pi(3^2)}$$

$$= \frac{630}{9}$$

$$= 70 \text{ cm}$$