End of Year Examination Paper 2

INSTRUCTION TO CANDIDATES:

1. Answer **all** questions.
2. Write your answers and working in the spaces provided.
3. Omission of essential working will result in loss of marks.
4. Calculators may be used in this paper.
5. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.

Marks Obtained
/80

Duration: 2 hours

1 Mr Gan is renovating his house which costs \$65 000. His savings will cover 30% of the amount and he will have to take a renovation loan on the remainder.

(a) Find the amount of loan that Mr Gan will need to take. [2]

The following information shows the different renovation loan packages that Mr Gan can take from two banks.

Bank *A*: Compound interest of 3% per annum compounded yearly, repayment period of 5 years.
Bank *B*: Simple interest of 4% per annum, repayment period of 5 years.

(b) Find the total amount of repayment if he takes the loan from

(i) Bank *A*, [2]

(ii) Bank *B*. [2]

(c) Calculate the monthly instalment Mr Gan needs to pay if he repays his loan with Bank *B* in equal monthly instalments. [2]

2 (a) Solve $\frac{3}{y^2} - \frac{5}{y^2 - 7} = 0$. [2]

(b) If the solutions of the equation $x^2 + ax + b = 0$ are -1 and 6, find the values of a and b. [3]

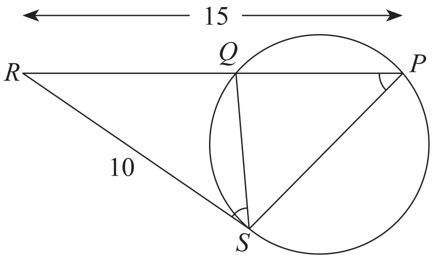
3 Mr Goh earns \$1200 more than Mr Suresh. $\frac{1}{2}$ of Mr Goh's salary is less than $\frac{3}{4}$ of Mr Suresh's salary. Their total salary is not more than \$16 000. Given that their salaries are whole numbers,

(a) write down two inequalities to represent the above information and solve the inequalities, [4]

(b) find the greatest possible salary for Mr Suresh, [1]

(c) find the least possible salary for Mr Goh. [1]

4 In the diagram, *RS* is a tangent to the circle at *S*, $\angle RPS = \angle QSR$ and *PQR* is a straight line.

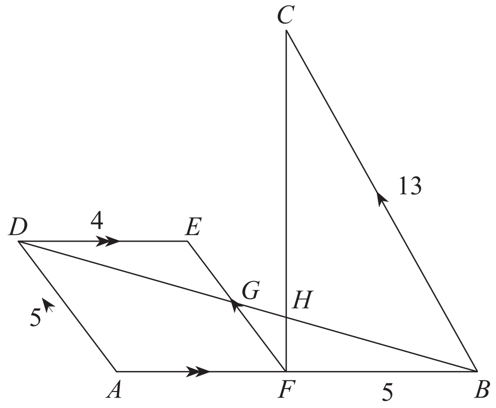


(a) Prove that triangle *RSP* is similar to triangle *RQS*. [3]

(b) Given that $RS = 10$ cm and $PR = 15$ cm, calculate the length of *QR*. [2]

(c) Given further that the area of triangle *RSP* is 48 cm^2, calculate the area of triangle *RQS*. [2]

5 In the figure, *AFED* is a parallelogram where *AB*//*DE* and *AD*//*FE*//*BC*. It is given further that *AD* = *BF* = 5 cm, *DE* = 4 cm, *BC* = 13 cm and ∠*BFC* = 90º. *DGHB*, *AFB* and *FHC* are straight lines.



(a) Name three pairs of similar triangles. [3]

(b) Calculate the length of

(i) *CF*, [2]

(ii) *EG*, [3]

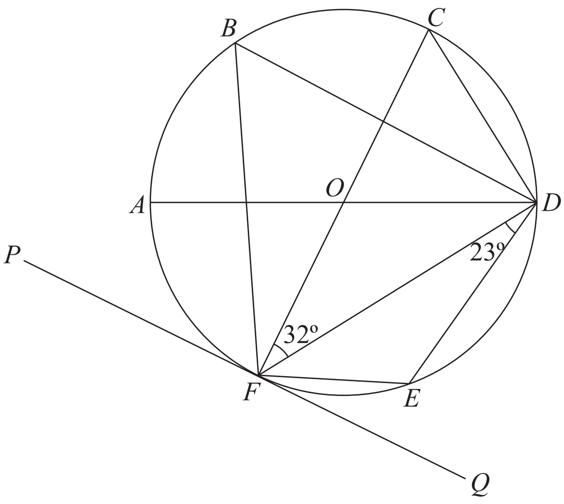
(iii) *AB*. [2]

(c) Find the value of the following fractions.

(i) $\dfrac{\text{Area of } \triangle DEG}{\text{Area of parallelogram } AFED}$ [1]

(ii) $\dfrac{\text{Area of } \triangle FGH}{\text{Area of } \triangle BCH}$ [1]

6 In the diagram below, A, B, C, D, E and F are points on the circle with centre O. AD and FC are the diameters of the circle and PFQ is a tangent to the circle at F. $\angle DFC = 32°$ and $\angle EDF = 23°$.



(a) Explain why CD is perpendicular to DF. [1]

(b) Calculate

(i) $\angle FCD$, [1]

(ii) $\angle FBD$, [1]

(iii) $\angle FOD$, [1]

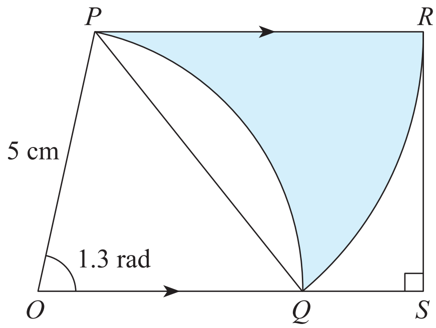
(iv) $\angle FED$, [1]

(v) $\angle EFQ$. [1]

(c) Deduce whether quadrilateral $FODE$ is a parallelogram. [2]

(d) Given that the diameter of the circle is 8 cm, find the area of $\triangle FCD$. [2]

7 In the diagram, PQ is an arc of a circle with centre O such that $\angle POQ = 1.3$ rad. PR is parallel to OS and $OP = 5$ cm. QR is an arc of another circle with centre P. RS is a tangent to the arc at point R where $\angle OSR = 90°$.



(a) Show that angle $QPR = 0.921$ radians. [2]

(b) Calculate the

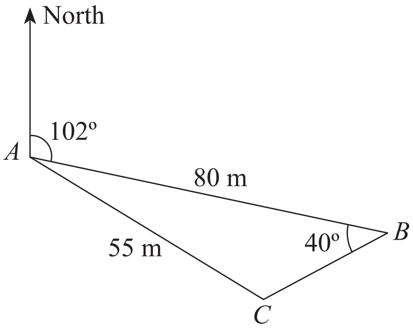
(i) length of chord PQ, [2]

(ii) perimeter of the shaded region, [3]

(iii) area of the shaded region, [3]

(iv) area of trapezium $PRSQ$. [2]

8 Three scouts, A, B and C are on a field. The bearing of scout B from A is 102º, $\angle ABC = 40º$, $AB = 80$ m and $AC = 55$ m.



(a) Find

(i) the bearing of C from B, [2]

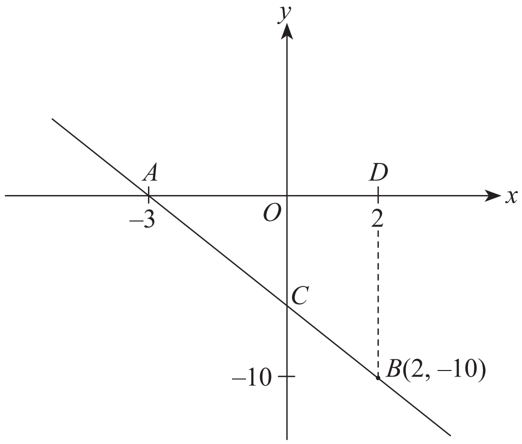
(ii) $\angle ACB$, given that ACB is an obtuse angle, [3]

(iii) the length of BC, [2]

(iv) area of $\triangle ABC$. [1]

(b) A bird flies 30 m directly above scout A. Find the angle of elevation of the bird from scout B, given that his eye level is 1.4 m above ground level. [2]

9 A straight line passes through the point $A(-3, 0)$ and the point $B(2, -10)$. It cuts the y-axis at the point C. $D(2, 0)$ is a point on the x-axis.



(a) By using similar triangles, find the y-coordinate of C. [2]

(b) Find the gradient of the line AB. [2]

(c) Find the equation of the line parallel to AB which passes through the origin. [2]

(d) Calculate the length of the line segment AB. [2]

(e) By calculating the area of $\triangle ADB$ or otherwise, find the perpendicular distance from D to AB. [2]

(f) The line segment AB is reflected on the x-axis to become AB_1. Find the equation of line AB_1. [2]

Solutions to:

End of Year Examination Paper 2

1. (a) Amount of loan $= \frac{70}{100} \times 65\,000$

$= \$45\,500$

(b) (i) Total repayment amount to Bank A

$= 45\,500\left(1 + \frac{3}{100}\right)^5$

$= \$52\,746.97$ (nearest cent)

(ii) Interest paid to Bank A $= \frac{45\,500 \times 4 \times 5}{100}$

$= \$9100$

Total repayment amount to Bank B

$= \$45\,500 + \$9100$

$= \$54\,600$

(c) Monthly installment $= \$54\,600 \div (5 \times 12)$

$= \$910$

2. (a)

$$\frac{3}{y^2} + \frac{5}{y^2 - 7} = 0$$

$$\frac{3(y^2 - 7) + 5y^2}{y^2(y^2 - 7)} = 0$$

$$3(y^2 - 7) + 5y^2 = 0$$

$$3y^2 - 21 + 5y^2 = 0$$

$$8y^2 = 21$$

$$y^2 = 2.625$$

$$y = \pm\sqrt{2.625}$$

$= -1.62$ or 1.62 (3 s.f.)

(b) An equation with roots -1 and 6, with x^2 coefficient $= 1$ is given by

$(x + 1)(x - 6) = 0$

$x^2 - 6x + x - 6 = 0$

$x^2 - 5x - 6 = 0$

By comparing coefficients with $x^2 + ax + b = 0$,

$a = -5$, compare x-coefficients

$b = -6$. compare constants

3. (a) Let Mr Goh's salary be $\$x$.

$\therefore$ Mr Suresh's salary is $\$(x - 1200)$.

$\frac{1}{2}x < \frac{3}{4}(x - 1200)$ and $x + (x - 1200) \leqslant 16\,000$

$\frac{1}{2}x < \frac{3}{4}x - 900$ and $x + x - 1200 \leqslant 16\,000$

$-\frac{1}{4}x < -900$ and $2x \leqslant 17\,200$

$x > 3600$ and $x \leqslant 8600$

Hence, $3600 < x \leqslant 8600$

(b) Greatest possible salary $= \$8600 - \$1200$

$= \$7400$

(c) Since Mr Goh's salary is a whole number and $x > 3600$,

least possible salary $= \$3601$

4. (a) $\angle SRP = \angle QRS$ (common $\angle$)

$\angle RPS = \angle RSQ$ (given)

$\therefore \triangle RSP$ is similar to $\triangle RQS$ (by AA).

(b) Since $\triangle RSP$ is similar to $\triangle RQS$,

$\frac{QR}{SR} = \frac{RS}{RP}$ corr. sides are proportional

$\frac{QR}{10} = \frac{10}{15}$

$QR = 6\frac{2}{3}$ cm

(c) Let the area of $\triangle RQS$ be A cm².

$\frac{A}{48} = \left(\frac{10}{15}\right)^2$ $\quad \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{A}{48} = \left(\frac{2}{3}\right)^2$

$\frac{A}{48} = \frac{4}{9}$

$A = 21\frac{1}{3}$ cm²

The area of $\triangle RQS$ is 21.3 cm².

5. (a) $\triangle BCH$ and $\triangle GFH$

$\triangle DEG$ and $\triangle BFG$

$\triangle BAD$ and $\triangle BFG$

(b) (i) Using Pythagoras' Theorem on $\triangle BFC$,

$CF^2 = 13^2 - 5^2$

$= 144$

$CF = \sqrt{144}$

$= 12$ cm

(ii) Since $\triangle DEG$ is similar to $\triangle BFG$,

$\frac{DE}{BF} = \frac{EG}{FG}$

$\frac{4}{5} = \frac{EG}{5 - EG}$

$4(5 - EG) = 5EG$

$20 - 4EG = 5EG$

$20 = 9EG$

$\therefore EG = 2\frac{2}{9}$ cm

(iii) $GF = 5 - 2\frac{2}{9}$

$= 2\frac{7}{9}$ cm

Since $\triangle BAD$ and $\triangle BFG$ are similar,

$\frac{BF}{AB} = \frac{GF}{DA}$

$\frac{5}{AB} = \frac{2\frac{7}{9}}{5}$

$\therefore AB = 9$ cm

(c) (i) $\frac{\text{Area of } \triangle DEG}{\text{Area of parallelogram } AFED} = \frac{\frac{1}{2} \times EG \times \cancel{h}}{EF \times \cancel{h}}$

$= \frac{\frac{1}{2} \times 2\frac{2}{9}}{5}$

$= \frac{2}{9}$

(ii) $\dfrac{\text{Area of } \triangle FGH}{\text{Area of } \triangle BCH} = \left(\dfrac{GF}{BC}\right)^2$

$= \left(\dfrac{2\frac{7}{9}}{13}\right)^2$

$= \dfrac{625}{13\ 689}$

6. (a) This is because CF is the diameter and the angle at the circumference subtended by the diameter is a right angle.

(b) (i) $\angle CDF = 90^\circ$ (rt. $\angle$ in semicircle)

$\angle FCD = 180^\circ - 90^\circ - 32^\circ$ ($\angle$ sum of $\triangle CDF$)

$= 58^\circ$

(ii) $\angle FBD = 58^\circ$ ($\angle$s in the same segment)

(iii) $\angle FOD$

$= 58^\circ \times 2$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 116^\circ$

(iv) $\angle FED = 180^\circ - 58^\circ$ ($\angle$s in opp. segments)

$= 122^\circ$

(v) $\angle EFD = 180^\circ - 23^\circ - 122^\circ$ ($\angle$ sum of $\triangle FED$)

$= 35^\circ$

$\angle EFQ = 90^\circ - 32^\circ - 35^\circ$ $\because \angle OFQ = 90^\circ$

$= 23^\circ$

(c) $\angle FOD = 116^\circ$ and $\angle FED = 122^\circ$

$\therefore \angle FOD \neq \angle FED$

Since the opposite angles of quad. $FODE$ are not equal, $FODE$ is not a parallelogram.

(c) $FC = 8$ cm

In $\triangle FCD$,

$\cos 32^\circ = \dfrac{FD}{8}$

$FD = 8 \cos 32^\circ$

Area of $\triangle FCD = \frac{1}{2}(8)(8 \cos 32^\circ)(\sin 32^\circ)$

$= 14.4 \text{ cm}^2$ (3 s.f.)

7. (a) $OP = OQ$ = radius of arc PQ

$\angle OQP = \dfrac{\pi - 1.3}{2}$ (base $\angle$ of isos. $\triangle$)

≈ 0.920796 rad

$\angle QPR = \angle OQP$ (alt $\angle$s, $PR//OS$)

$= 0.921$ rad (3 s.f.)

(b) (i) Using the Cosine Rule on $\triangle OPQ$,

$PQ = \sqrt{5^2 + 5^2 - 2(5)(5)\cos 1.3}$ radian mode

$PQ \approx 6.05186$ cm

$= 6.05$ cm (3 s.f.)

(ii) Length of arc $PQ = 5(1.3)$ $s = r\theta$

$= 6.5$ cm

Arc QR is subtended by $\angle QPR$ at the centre, where radius $= PQ$.

Length of arc $QR = (6.05186)(0.921)$

≈ 5.57376 cm

Perimeter of shaded region

$= PR + \text{arc } QR + \text{arc } PQ$

$= 6.05186 + 5.57376 + 6.5$

$= 18.1$ cm (3 s.f.)

(iii) Area of segment PQ

$= \frac{1}{2}(5^2)(1.3 - \sin 1.3)$ $\frac{1}{2}r^2(\theta - \sin\theta)$

$\approx 4.20552 \text{ cm}^2$

Area of sector $PQR = \frac{1}{2}(PQ^2)(\angle QPR)$

$= \frac{1}{2}(6.05186^2)(0.921)$

$\approx 16.8658 \text{ cm}^2$

Area of shaded region

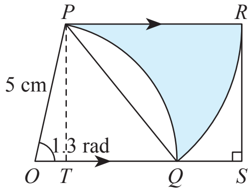
= Area of sector PQR – Area of segment PQ

$= 16.8658 - 4.20552$

$\approx 12.66028 \text{ cm}^2$

$= 12.7 \text{ cm}^2$ (3 s.f.)

(iv)



Draw a perpendicular from P to OS.

In $\triangle OPT$,

$\cos 1.3 = \dfrac{OT}{OP}$

$= \dfrac{OT}{5}$

$OT \approx 1.33749$ cm

Similarly,

$\sin 1.3 = \dfrac{PT}{OP}$

$= \dfrac{PT}{5}$

$PT \approx 4.81779$ cm

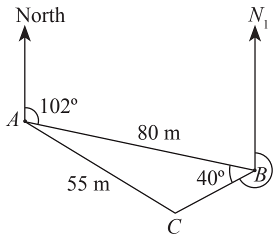
Area of trapezium

$= \frac{1}{2}(PR + OS)(PT)$

$= \frac{1}{2}[6.05186 + (1.33749 + 6.05186)](4.81779)$

$= 32.4 \text{ cm}^2$ (3 s.f.)

8. (a) (i) Draw a North line at B.



$\angle N_1BA = 180^\circ - 102^\circ$ (int. $\angle$s, $AN//BN_1$)

$= 78^\circ$

Bearing of C from $B = 360^\circ - 78^\circ - 40^\circ$ ($\angle$s at a pt.)

$= 242^\circ$

(ii) Using the Sine Rule on $\triangle ABC$,

$$\frac{80}{\sin \angle ACB} = \frac{55}{\sin 40^\circ}$$

$$\sin \angle ACB \approx 0.93496$$
$$\angle ACB \approx 69.222 \text{ or } 180^\circ - 69.222$$
$$= 69.222^\circ \text{ (rej.) or } 110.778^\circ$$

$\therefore$ Obtuse $\angle ACB = 110.8^\circ$ (1 d.p.)

(iii) $\angle BAC$

$= 180^\circ - 110.778^\circ - 40^\circ$ ($\angle$sum of $\triangle ABC$)

$= 29.222^\circ$

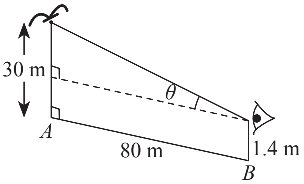
Using the Cosine Rule on $\triangle ABC$,

$$BC^2 = 80^2 + 55^2 - 2(80)(55)\cos 29.222^\circ$$
$$= 1744.9347$$
$$BC = \sqrt{1744.9347}$$
$$\approx 41.772 \text{ cm}$$
$$= 41.8 \text{ cm}$$ (3 s.f.)

(iv) Area of $\triangle ABC = \frac{1}{2}(41.772)(80)\sin 40^\circ$

$= 1070 \text{ m}^2$ (3 s.f.)

(b)



Let the angle of elevation be θ.

$$\therefore \tan\theta = \frac{30 - 1.4}{80}$$
$$\tan\theta = 0.3575$$
$$\theta = 19.7^\circ$$ (1 d.p.)

The angle of elevation is 19.7°.

9. (a) $\triangle AOC$ and $\triangle ADB$ are similar triangles.

$$\frac{AO}{AD} = \frac{OC}{DB}$$ corr. sides are proportional

$$\frac{3}{5} = \frac{OC}{10}$$

$OC = 6$

$\therefore$ y-coordinate of C is –6.

(b) Gradient of $AB = \dfrac{0-(-10)}{-3-2}$

$= -2$

(c) When the line passes through the origin,

y-intercept = 0.

$\therefore$ The equation of the line is $y = -2x$.

(d) Using Pythagoras' Theorem on $\triangle ADB$,

$$AB = \sqrt{(-3-2)^2 + [0-(10)]^2}$$
$$= \sqrt{125} \text{ units}$$
$$= 11.2 \text{ units}$$ (3 s.f.)

(e) Area of $\triangle ADB = \frac{1}{2} \times AD \times BD$

$= \frac{1}{2} \times 5 \times 10$

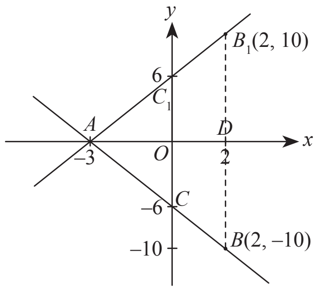
$= 25 \text{ units}^2$

Let the perpendicular distance from D to AB be h units.

Equating the area of $\triangle ADB$,

$$\frac{1}{2} \times AB \times h = 25$$
$$\frac{1}{2} \times \sqrt{125} \times h = 25$$
$$h = 4.47 \text{ units}$$ (3 s.f.)

(f)



B is reflected to become $B_1(2, 10)$.

C is reflected to become $C_1(0, 6)$, which is also the y-intercept of the reflected line.

Gradient of $AB' = \dfrac{10-0}{2-(-3)}$

$= \dfrac{10}{5}$

$= 2$

$\therefore$ The equation of line AB_1 is $y = 2x + 6$.