Activity 1

SURFACE AREA

Discussion

Question 1

- Students need to visualize how the items are stacked, determine which surfaces are on the outside and calculate the sum of all the areas.
- Care will be needed to ensure all units used are the same (metres expressed as decimals or fractions in centimetres)

Question 2

- When the 3 × 3 cube is considered, there will be one 1 cm^3 in the middle – when it is removed to create a hole all the way through the cube, the surface area painted will be 6 cm^2 less.

Question 3

- The 4 × 4 cube will have a 2 × 2 cube in the middle – the surface area will be 24 cm^2 less when this is removed to make the hole.

Question 4

- There are different possibilities for the 5 × 5 cube according to whether the hole is 1 cm × 1 cm or 3 cm × 3 cm when the cubes are removed to make the same hole from all 6 sides. Students may need to use cubes to see what is happening.

Activity 2

VOLUME AND SURFACE AREA

Discussion

Question 1

- Students are encouraged to see 3-dimensional solids in terms of the component cubes and work out the surface area by finding the number of exposed faces and the volume by working out the total number of cubes.

Question 2(b)

- The cubes vary in size, so other factors also come into the calculations.
- It is helpful to work directly from a diagram that shows how the cubes are arranged and from plans that use a grid to indicate the number of cubes in each position.
- If possible, actual cubes can be used to visualise the various shapes.

Activity 3

SQUARES AND AREA

Discussion

Question 1(a)

- Students may use a tangram set to manipulate the component pieces and see the relationship of parts to wholes.

Question 1(e)

- At first the problem may seem impossible, as the square cannot simply be placed inside the large triangle to directly determine its area. Instead, the problem needs to be broken down into smaller problems that together will answer the initial question.

Activity 5

PROFIT AND LOSS

Discussion

Question 1

- Students are required to read and interpret information and use this information to find a solution.
- Students need to think in terms of fixed costs occurring regardless of income and variable costs occurring according to production.
- The various cost can be recorded in a table.

Question 2

- Once the table is completed, the data can then be used to find when the factory will move from making a loss to making a profit.

Question 3

- Students will be able to find where the break-even point occurs from the table.

Activity 7

PUZZLE SCROLLS (1)

Discussion

Question 2

- For the second scroll exploring the number of beads, students can think in terms of multiples. If they find 13 by 8 (the first 3-digit multiple of 8), which is 104, add one to this number to give the starting number. A calculator can also be used (add 8, press the equal key and it will add 8 each time). These numbers can be recorded and students can then look at multiples of 7 (with 1 added) and highlight any numbers that are the same and then, from these possibilities, check to see if it is also true for 6. (*)

Question 3

- For the third scroll involving 3-digit numbers adding to 6, the possible solutions from numbers in one hundred can be used to determine other numbers in 2 hundred and so on.

Question 5

- For the fifth scroll exploring 3-digit numbers when divided by 5 and 7, it will mean that each number has to end in a 3 or 8. The first 3-digit number divisible by 7 is 105. If we add 2 to this number, we get 107, which can be our starting number for looking for other numbers. If we key 107 into a calculator, add 7 and press equal, it will keep adding 7 each time. If we then record any numbers that end in 3 or 8, we have the numbers that will result in a remainder of 3 when divided by 5 and a remainder of 2 when divided by 7. (*)

(*) This function may not work for some scientific calculators.

Activity 9

WILDERNESS EXPLORER

Discussion

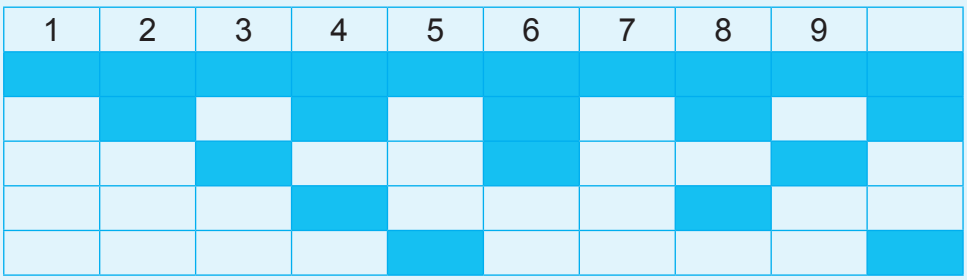
- Students read the information on the page and use it to find a number of solutions.
- Students who are not familiar with 24-hour time can still do the activity using a conversion table.
- The investigation can be used to explore the concept of 24-hour time.
- Students need to read and extract information from a number of sources and fit this information against set criteria.
- There are a number of flights that fit the criteria, with other flights resulting in the traveller getting to Brinchang either too early or too late. Once a flight has been deemed too late, then others that are later can be automatically excluded. For example, if the Carrier Airlines flight at 14 25 will not get you to Brinchang before dinner, then the flights at 15 50 and 16 45 can also be ruled out. No further explorations are needed for these flights.
- Similar thinking can be used for the flights that are too early. If catching the Temasek Airlines flight at 06 20 gets you to the campsite too early, then all flights prior to this time will also be too early and can be quickly excluded.
- Students need to think in terms of 24-hour time for the flight information, but the before and after times of 1 pm and 6.30 pm are in 12-hour time. They also need to take into consideration that 1 pm and 6.30 pm are 13 00 and 18 30 respectively.
- The information regarding the taxi and the waiting time at the airport is needed for the last investigation to determine what the latest time to leave home to go to the airport would be. Some students may try to include this in their travel calculations.
- The shuttle bus leaves the airport every hour starting at 07 20, which means a shuttle bus leaves 20 past each hour. However, if the plane arrives at 11 15, then they will not be able to catch the 11 20 shuttle bus as they need to exit the plane and collect their luggage. Students need to consider this time in their solutions.

Activity 10

CHANGING LOCKERS

Discussion

- The first part of the activity can be solved using counters on a grid or colouring the squares to see what is happening.



The number of possibilities can then be seen directly or patterns can be sought.

- Analysis of the patterns shows that the squares represent the factors of the number of the column in which they occur.
- The other questions are solved by considering prime numbers, squares of prime numbers and systematically examining the pairs of factors in each number.

Activity 16

NUMBER PATTERNS

Discussion

Question 1

- Students should use different coloured counters to explore the pattern shown on the page and extend it to larger numbers in order to describe the relationship between the counters and the triangle.
- The sum of the first two numbers, 1 + 2, is 3. The sum of the first three numbers, 1 + 2 + 3, is 6. The sum of the first four numbers, 1 + 2 + 3 + 4, is 10, and so on.
- Two of the same triangular numbers give the corresponding square number and the number, showing a given triangular number is half of [number^2 + number].
- Students should use their counters to investigate how this pattern continues for larger triangular numbers, introducing the use of T_1, T_2, T_3, …
- Another pattern that students might see is that the triangular number is half of [number × (number + 1)].
- When two consecutive triangular numbers are added, they give the square number corresponding to the larger triangular number. This is readily shown with counters for a series of numbers and can be expressed as $T_2 + T_3 = S_3$, $T_3 + T_4 = S_4$ and so on.

Question 2

- This arrangement of numbers in a triangular pattern used by Pascal, in fact first suggested by ancient Chinese mathematicians, is very helpful for summarising relationships in probability (Pascal) and algebra (Ancient China). There are also many interesting patterns that can be discerned among the numbers. The outside diagonals of the triangle consist only of 1, the next diagonal has the counting numbers, and the triangular numbers are in the third diagonal, beginning with 3 (T_2). When the triangular numbers are added, their sum is diagonally below the last number.
- Many other patterns can be investigated as the triangle is extended. Teachers can have students write out the numbers on a sheet of A4 paper to get as large a triangle as they can, and ask them to highlight the triangular and square numbers in the triangle.
- Explore questions such as:

– Is it possible to have a square triangular number?
– Is there a pattern to the location of the square numbers?
– Can they find a way to describe a pattern for the numbers in the other diagonals?

Activity 19

ALPHAMETIC PUZZLES

Discussion

- These puzzles are known as alphametic or cryptorithm puzzles where letters in words are substituted for numbers in an addition algorithm.
- There are a number of famous alphametic puzzles such as ‘send more money’ and ‘no more cash’. Both of these puzzles have a number of different possibilities that can fit the criteria.
- In the example provided, you know that M must be 1 as you are adding two 4-digit numbers and as such can only result in a 1 in the ten thousands place. As M and S are different letters, you know that S must be 8 or 9 to result in a 5-digit number. Either way, O must be 0. If you decided to make S a 9 and look to the hundreds place, the two unknown digits must be consecutive as we know that O is 0 and as the other letters are different, a ten must have been renamed to get a different letter when adding zero. At this stage, it is possible to use a number of different combinations. We can see that 4 and 5 work as do 5 and 6. Once you select your consecutive numbers and have found E, you can have a number in both the tens and the ones places to work around.
- Students can be encouraged to see how many different possibilities there are.
- Teachers may like to give students a number in each puzzle to get started (e.g. E is 5).

Activity 20

AT THE SHOPS

Discussion

- The 'shopping' questions have more than one step with a number of operations and centre around discounts and sale prices.
- The concepts of marked price and discount need to be explored in order for students to come to terms with what the questions are asking.
- Some questions involve calculating the price after the discount while others involve calculating the full price.
- A table can be used to assist with the questions involving a set amount and investigating what could be bought for that amount.
- A calculator can also be used to assist if necessary.

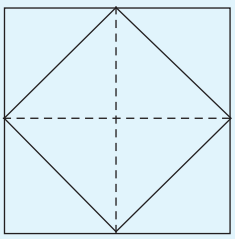
Activity 27

MAKING DESIGNS

Discussion

Question 1

- Students need to visualise the way in which the squares are reducing each time to be half of the one in which they are embedded.
- Since the area of the original square is 1600 cm^2, the next square has an area of 800 cm^2, the next 400 cm^2, the next 200 cm^2, and the innermost square has an area of 100 cm^2.



Activity 29

DESIGNER SQUARES

Discussion

- Rather than using complex calculations based on areas of triangles, visualising how the diamonds shape can be used to form a larger triangle, which is half of a square, to allow students to find the area of the garden bed.
- Students will find it helpful to use triangles to make two of the diamonds and they can rearrange them to see that they form a large square.

Activity 30

TRAVELLING TO AUSTRALIA

Discussion

Question 1

- Models showing the different months can be used as a starting point. The problem states how many visitors there were during August and this information can be used to work out the number of visitors in July – 9000 more. That information can in turn be used to work out the numbers for the other months.

Questions 2 & 3

- The number of visitors in 2006 is not needed to find the solution. The last problem contains additional information about an increased percentage of visitors, which is also not needed to find the solution.

Activity 31

DISTANCE

Discussion

Question 1

- This problem states that Paula runs 100 m in 'about 50 seconds'. This is not an exact time and varies from lap to lap, so the solution of how far she has run will again be an approximate distance.

Question 4

- This investigation contains information about morning tea and lunch that needs to be factored into the amount of time driving.

Question 5

- This problem deals with the concept of distance over a month. Discussion can centre on how this varies from month to month.

Activity 32

COST

Discussion

Question 1

- This investigation requires students to calculate how many 250-g packs can be made from 74 kg of cheese and how many 200-g packs can be made from 42 kg of sun-dried tomatoes, and to use this to calculate how much profit is made across a year.

Question 2

- It is important to factor in that Mal is working on both Saturday and Sunday, and thus getting paid at $47.50 per day instead of $32.75.

Question 4

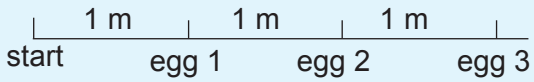
- It is important to keep in mind that each bracket needs six screws, which come in packs of 10. Students need to calculate how many packets rather than how many screws are required.

Activity 33

PROSPECT PLAINS

Discussion

- This activity explores problems with large amount of data that need both computation and patterning to determine solutions.
- In these problems, the information needs to be carefully analysed to determine the number of eggs that are set out for the race as well as the distance to run to pick up and return the eggs to the basket, collecting two eggs at a time.
- Students may use counters or a diagram to represent the race and see what is happening.



Activity 37

RIDING TO WORK

Discussion

Question 1

- In this problem, Gary is halfway around the track and cycles 600 m in 1 minute. Geoff is one-third of the way around the track and cycles 400 m in 1 minute. Exploring how far each rider has cycled after 1, 2, 3, … minutes shows that both cyclists are at the start after 6, 12, 18, … minutes.
- Another way is to see that Gary reaches the start every 2 minutes, Geoff every 3 minutes – 6, 12, 18, … are common multiples of 2 and 3.

Question 3

- Analysing this problem shows that Geoff rides $\frac{24}{56}$ or $\frac{3}{7}$ of the distance around the track before they meet. Gary must have cycled $\frac{4}{7}$ of the distance in 24 seconds. He would take 6 seconds to travel $\frac{1}{7}$ of the distance and 42 seconds to complete one lap. Put this information into a table.

Activity 38

BIKE TRACKS

Discussion

Question 1

- This problem is solved the same way as in Activity 37, only the fractions involved are now $\frac{4}{9}$, $\frac{8}{9}$, $\frac{12}{9}$, … and $\frac{5}{9}$, $\frac{10}{9}$, $\frac{15}{9}$, … .

Question 5

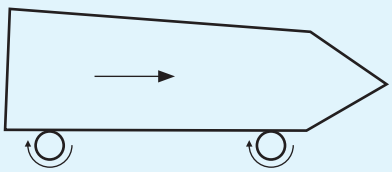
- This problem can be solved by placing some of the information in a table to see the pattern, but 98 entries is too many. Alternatively, use the different intervals of 4, 5, 2 and 3 days to get a common multiple of 60.

Activity 43

ROLLING ALONG

Discussion

- It will be helpful for students to first model the action of the obelisk moving along with the aid of rollers.
- If a block is set up on two or more cylinders touching each other and the point of contact of the block and the front cylinder are marked, it will be possible to see how the block moves forward on the cylinders at the same time as the cylinders move along the ground.
- For each complete turn of the rollers, the obelisk must move a distance equal to the circumference along the rollers and a further distance equal to the circumference along the ground.



- The experience with the rollers will help in imagining how the gears move – when Gear A moves in an anticlockwise direction, Gear B will move in a clockwise direction and Gear C in an anticlockwise direction.

Activity 44

BALANCING OUT

Discussion

- The problems in this activity are essentially solved by ruling out different possibilities one at a time.
- Students are encouraged to look for an efficient means of solving the problems rather than the tedious task of comparing the weights of the coins one at a time.
- In this way, they can come to terms with the idea of creating three distinct groupings of the coins (or gold bars) to quickly determine which group contains the lighter or heavier object and then repeat the process until they can identify which coin is lighter.

Activity 47

DISTANCE TRAVELLED

Discussion

Question 1

- For this question, Kevin needs to walk as far as the second, fourth, sixth (and so on) tree and back to the start. However, he can finish watering at the last tree and does not need to return to the tap. (Next time, he can do the same in reverse, starting from the last tree.)

Question 2

- Clearly, the answer to this question is not that the cyclists returned at the same time! A diagram or use of counters sorts out the difficulty posed by the question.

Question 3

- In this question, the information about the ascent and descent of the hot air balloon is not relevant. Rather, students need to understand that the path traces a rectangle because the directions are at right angles to one another and show that the final point is one side of the rectangle.

Activity 50

SOCCER RECORDS

Discussion

- The questions require a clear understanding of how pages in a book are numbered.
- Each side of one sheet of paper that makes up a book is one page. This may lead some students to divide the total number of stickers/digits that are used by two. This could not be correct as it assumes that each page would be numbered with a 1-digit number and there are clearly more than nine pages.
- There will be pages that need two stickers as they are numbered with two-digit numbers and there are also some that use three-digit numbers and need three stickers.
- Some form of organised list or table is needed to keep track of all the information or attempts that are tried and adjusted.
- One way is to focus on the number of digits on a page, the number of pages with that number of digits and the number of stickers this requires:

number of digits on a page	number of pages	number of stickers used
1	9	9
2		
3		

Activity 51

BUILDING HOUSES

Discussion

Question 1

- This question requires a careful analysis of all the information presented, as well as a sense of proportion to see that the first task would be completed in less time whereas the second one would take more time. While a week has seven days, a working week varies according to the type of work and the expectations of the workers.

Question 2

- This question is similar, but this time some thought about what is involved in the problem would suggest that more time is needed. Care needs to be taken that this group of workers only works five days per week.

Question 3

- This question is also similar, but there are now more pieces of information to keep in mind. It has to take less time, hence there must be more workers.