Algebraic Manipulation and Formulae

1. Express each of the following as a single fraction in its simplest form.

(a) $\dfrac{6}{x-5}+\dfrac{4}{3x-15}$

(b) $\dfrac{8x}{6+x-x^2}-\dfrac{2x}{3-x}$

(c) $\dfrac{3}{x-1}-\dfrac{4}{3x+2}+\dfrac{5}{3x^2-x-2}$

2. Simplify $\dfrac{x^2-81-4xy+4y^2}{7x-14y+63}$.

3. Simplify $\dfrac{a^2+8ab+3a+12b+16b^2}{ac-3ad+4bc-12bd}$.

Solutions to

Algebraic Manipulation and Formulae

1. (a) $\dfrac{6}{x-5}+\dfrac{4}{3x-15}=\dfrac{6}{x-5}+\dfrac{4}{3(x-5)}$

$=\dfrac{3(6)+4}{3(x-5)}$

$=\dfrac{22}{3(x-5)}$

(b) $\dfrac{8x}{6+x-x^2}-\dfrac{2x}{3-x}=\dfrac{8x}{(3-x)(2+x)}-\dfrac{2x}{3-x}$

$=\dfrac{8x-2x(2+x)}{(3-x)(2+x)}$

$=\dfrac{8x-4x-2x^2}{(3-x)(2+x)}$

$=\dfrac{4x-2x^2}{(3-x)(2+x)}$

$=\dfrac{2x(2-x)}{(3-x)(2+x)}$

(c) $\dfrac{3}{x-1}-\dfrac{4}{3x+2}+\dfrac{5}{3x^2-x-2}=\dfrac{3}{x-1}-\dfrac{4}{3x+2}+\dfrac{5}{(x-1)(3x+2)}$

$=\dfrac{3(3x+2)-4(x-1)+5}{(x-1)(3x+2)}$

$=\dfrac{9x+6-4x+4+5}{(x-1)(3x+2)}$

$=\dfrac{5x+15}{(x-1)(3x+2)}$

$=\dfrac{5(x+3)}{(x-1)(3x+2)}$

2. $\dfrac{x^2-81-4xy+4y^2}{7x-14y+63}=\dfrac{x^2-4xy+4y^2-81}{7(x-2y+9)}$

$=\dfrac{(x-2y)(x-2y)-9^2}{7(x-2y+9)}$

$=\dfrac{[(x-2y)-9][(x-2y)+9]}{7(x-2y+9)}$

$=\dfrac{(x-2y-9)(x-2y+9)}{7(x-2y+9)}$

$=\dfrac{x-2y-9}{7}$

3. $\dfrac{a^2+8ab+3a+12b+16b^2}{ac-3ad+4bc-12bd}=\dfrac{a^2+8ab+16b^2+3a+12b}{a(c-3d)+4b(c-3d)}$

$=\dfrac{(a+4b)^2+3(a+4b)}{(c-3d)(a+4b)}$

$=\dfrac{(a+4b)(a+4b+3)}{(c-3d)(a+4b)}$

$=\dfrac{a+4b+3}{c-3d}$